JEE Advanced Test 1

DURATION: 0 Hours 30 Minutes **DATE: 2025-03-28**

SYLLABUS

Mathematics:	Trigonometric Functions, Principle Of Mathematical Induction, Binomial Theorem.
Physics:	Physical World And Measurements, Motion In A Plane, Laws Of Motion.
Chemistry:	Some Basic Concepts In Chemistry, Structure Of Atom.

(Mathematics)

1. Let $X=\left\{(x, y) \in \mathbb{Z} \times \mathbb{Z}: \frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

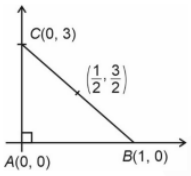
A) $\frac{71}{220}$ **B)** $\frac{73}{220}$

C) $\frac{79}{220}$ **D)** $\frac{83}{220}$

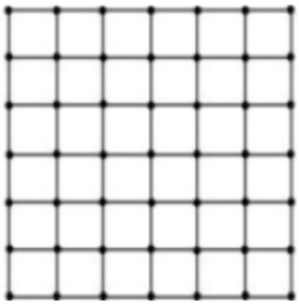
2. Let Q be the cube with the set of vertices $\left\{\left(x_1, x_2, x_3\right) \in \mathbb{R}^3: x_1, x_2, x_3 \in\{0,1\}\right\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices $(0,0,0)$ and $(1,1,1)$ is in S. For lines ℓ_1 and ℓ_2, let $d\left(\ell_1, \ell_2\right)$ denote the shortest distance between them. Then the maximum value of $d\left(\ell_1, \ell_2\right)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

A) $\frac{1}{\sqrt{6}}$ **B)** $\frac{1}{\sqrt{8}}$

C) $\frac{1}{\sqrt{3}}$ **D)** $\frac{1}{\sqrt{12}}$



3. Let $M=\left(a_{ij}\right), i, j \in\{1,2,3\}$, be the 3×3 matrix such that $a_{ij}=1$ if $j+1$ is divisible by i, otherwise $a_{ij}=0$. Then which of the following statements is(are) true?



A) M is invertible

B) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}=\begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

C) The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\} \neq \{\mathbf{0}\}$, where $\mathbf{0}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

D) The matrix $(M-2I)$ is invertible, where I is the 3×3 identity matrix

4. Let $f:[0,1] \rightarrow [0,1]$ be the function defined by $f(x)=\frac{x^3}{3}-x^2+\frac{5}{9}x+\frac{17}{36}$. Consider the square region $S=[0,1] \times [0,1]$. Let $G=\{(x, y) \in S: y>f(x)\}$ be called the green region and $R=\{(x, y) \in S: y \text{ be called the red region. Let } L_h=\{(x, h) \in S: x \in [0,1]\} \text{ be the horizontal line drawn at height } h \in [0,1]$. Then which of the following statement is(are) true?

A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

5. Let $n \geq 2$ be a natural number and $f:[0,1] \rightarrow \mathbb{R}$ be the function defined by f(x)=

$$\begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x=0, x=1, y=0$ and $y=f(x)$ is 4 , then the maximum value of the function f is

6. Let

$$R=\left\{\begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0,3,5,7,11,13,17,19\}\right\}$$

. Then the number of invertible matrices in R is

7.

Consider the following lists:

List-I	List-II
(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P) has two elements
(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q) has three elements
(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S) has five elements
	(T) has six elements

The correct option is:

A) (I) $\rightarrow$ (P); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (S)

B) (I) $\rightarrow$ (P); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (R)

C) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (S)

D) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)

8. Let p, q, r be nonzero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations

$$x + y + z = 1$$
$$10x + 100y + 1000z = 0$$
$$qrx + pry + pqz = 0$$

List-I	List-II
(I) If $\frac{q}{r} = 10$, then the system of linear equations has	(P) $x = 0,\ y = \frac{10}{9},\ z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) $x = \frac{10}{9},\ y = -\frac{1}{9},\ z = 0$ as a solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q} = 10$, then the system of linear equations has	(S) no solution
	(T) at least one solution

The correct option is:

A) (I) $\rightarrow$ (T); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (T)

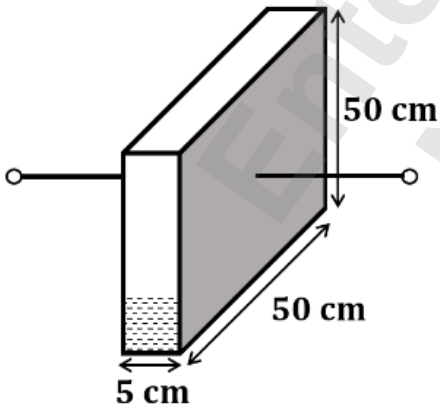
B) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)

C) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)

D) (I) $\rightarrow$ (T); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (T)

(Physics)

9. A container has a base of $50\ \text{cm} \times 5\ \text{cm}$ and height 50 cm , as shown in the figure. It has two parallel electrically conducting walls each of area $50\ \text{cm} \times 50\ \text{cm}$. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of $250\ \text{cm}^3\ \text{s}^{-1}$. What is the value of the capacitance of the container after 10 seconds? [Given: Permittivity of free space $\epsilon_0 = 9 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$, the effects of the non-conducting walls on the capacitance are negligible]



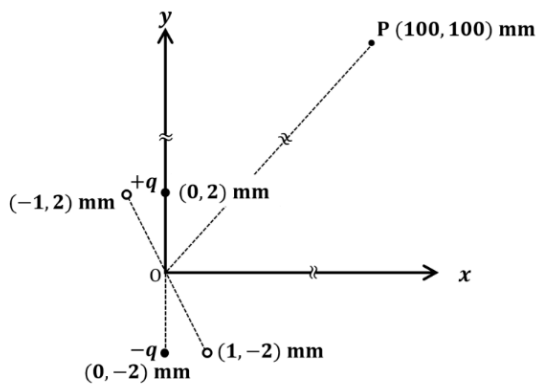
A) 27 pF

B) 63 pF

C) 81 pF

D) 135 pF

10. An electric dipole is formed by two charges $+q$ and $-q$ located in xy-plane at $(0, 2)\text{mm}$ and $(0, -2)\text{mm}$, respectively, as shown in the figure. The electric potential at point $\text{P}(100, 100)\text{mm}$ due to the dipole is V_0. The charges $+q$ and $-q$ are then moved to the points $(-1, 2)\text{mm}$ and $(1, -2)\text{mm}$, respectively. What is the value of electric potential at P due to the new dipole?



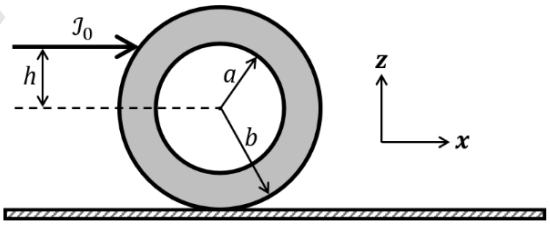
A) $V_0/4$

B) $V_0/2$

C) $V_0/\sqrt{2}$

D) $3V_0/4$

11. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $\mathcal{J}_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct?



A) For $\mu \neq 0$ and $a \rightarrow 0, h_m = b/2$.

B) For $\mu \neq 0$ and $a \rightarrow b, h_m = b$.

C) For $h = h_m$, the initial angular velocity does not depend on the inner radius a.

D) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

12. A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60^\circ$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{\text{min}} = 30^\circ$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?

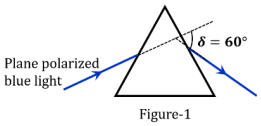


Figure-1

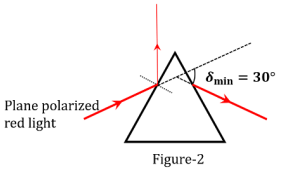


Figure-2

A) The blue light is polarized in the plane of incidence.

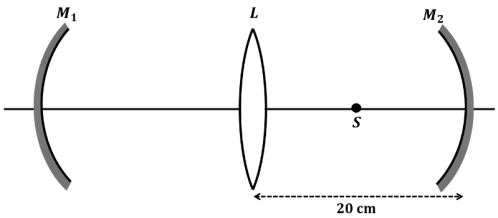
B) The angle of the prism is 45°.

C) The refractive index of the material of the prism for red light is $\sqrt{2}$.

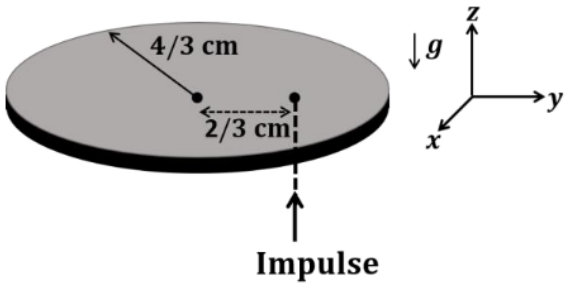
D) The angle of refraction for blue light in air at the exit plane of the prism is 60°.

13.

An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid-point between L and M_2 on the axis. When the distance between L and M_1 is $n/7$ cm, one of the images coincides with S. The value of n is _____.

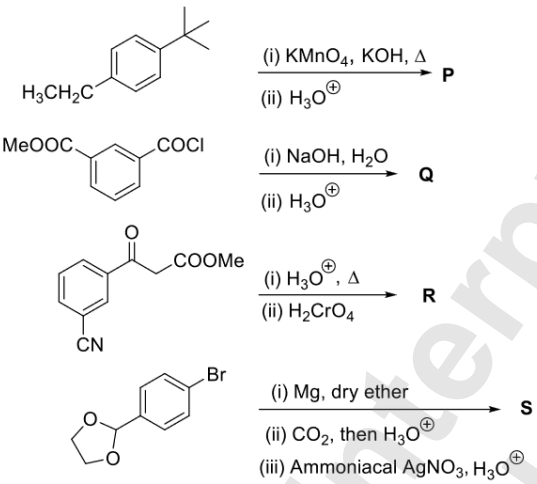


14. A thin circular coin of mass 5 gm and radius $4/3$ cm is initially in a horizontal xy-plane. The coin is tossed vertically up ($+z$ direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N-s at a distance $2/3$ cm from its center. The coin spins about its diameter and moves along the $+z$ direction. By the time the coin reaches back to its initial position, it completes n rotations. The value of n is [Given: The acceleration due to gravity $g = 10\ \mathrm{m\ s^{-2}}$]

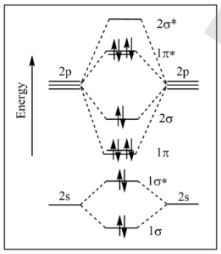


(Chemistry)

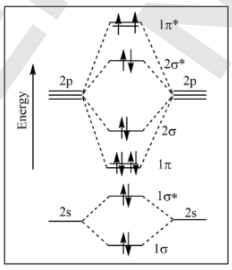
15. In the following reactions P, Q, R and S are the major products.



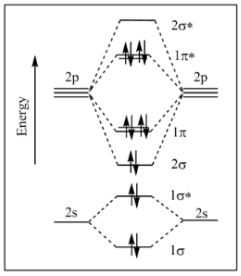
A)



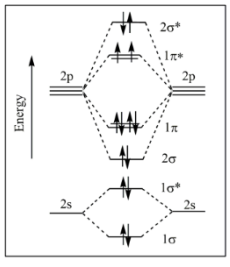
B)



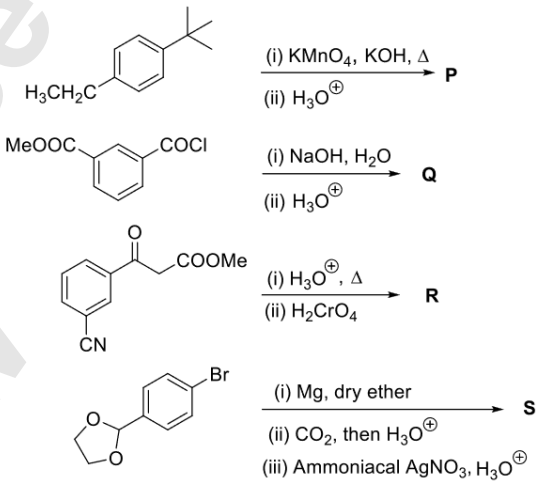
C)



D)



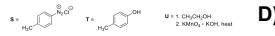
16. Considering the following reaction sequence,

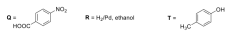


A) $\mathbf{P} = \mathrm{H_2/Pd}$, ethanol

\mathbf{R}=\mathrm{NaNO}_2 / \mathrm{HCl} \quad \mathbf{U}=1 . \mathrm{H}_3 \mathrm{PO}_2 2. $\mathrm{KMnO_4 - KOH}$, heat

B) 

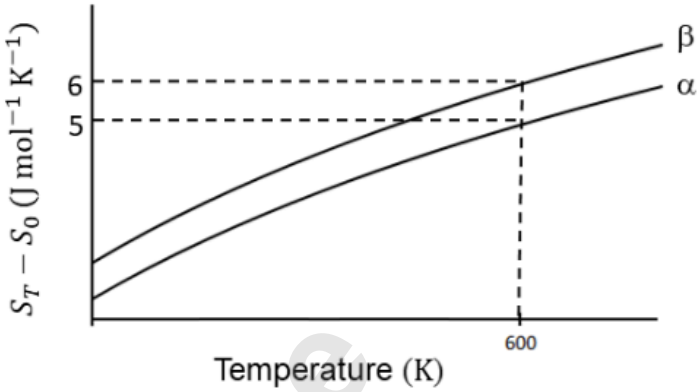
C) 

D) 

Paragraph :

"PARAGRAPH"

The entropy versus temperature plot for phases α and β at 1 bar pressure is given. S_T and S_0 are entropies of the phases at temperatures T and 0 K, respectively.



The transition temperature for α to β phase change is 600 K and $C_{p,\beta} - C_{p,\alpha} = 1$ J mol^{-1} K^{-1}. Assu
$(C_{p,\beta} - C_{p,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{p,\alpha}$ and $C_{p,\beta}$ are heat capaci
of α and β phases, respectively.

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17. The value of entropy change, $S_\beta - S_\alpha$ (in J mol^{-1} K^{-1}), at 300 K is . [Use: $\ln 2 = 0.69$ Given: $S_\beta - S_\alpha = 0$ at 0 K]

18. The number of heteroatoms present in one molecule of **R** is [Use: Molar mass (in gmol^{-1}): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5 Atoms other than C and H are considered as heteroatoms]